

The lives of the mathematicians

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If people do not believe that mathematics is simple, it is only because they do not realize how complicated life is.

John Louis von Neumann

Lesson 1

The reply seemed innocuous enough: a two bedroom apartment in the *Centrum*, each of 18 m². One could rent one room or both for double the price. No mention of being out of the country and thus unable to show you the apartment. No mention of posting you the keys once the deposit and the first rental payment are received. At least not yet. The level of the English was slightly subpar, but this was not all that strange, not in Amsterdam where people (even, probably, potential landlords) had first languages ranging from Lithuanian to Greek. However, it was clear this was a rental scam. It was clear because, fortunately, the scammer had been careless. David had replied to the first response with only a modicum of suspicion (the price of a room was just a little bit too good to be true, which means it was just barely reasonable rather than completely unaffordable). However, when he received a second response, to a different ad, in relation to a different apartment, with almost identical wording, his (now fast developing) *scam sense* tingled. When it turned out this second response was from the same person he was quite sure. This was the third scammer he had come across in less than a week. The first had nearly fooled him, but speaking to the tenants of neighbouring apartments had made it clear that the apartment being offered did not, in fact, even exist.

David gave a mental sigh, then went on to report the scammer to the relevant authorities and the websites where the ads had been placed. Finding a place to stay in Amsterdam was exhausting. And the state of constant frustration that it precipitated was not at all conducive to doing mathematics. David decided to give up the search for a new room for the day. He still had that problem to finish. He was not in the best state of mind for this. One needed to be calm, open-minded, fresh. In the absence of a truly fresh mind, David would need to settle for one soothed by the Baroque sounds of Vivaldi and Bach. The great mathematician G.H. Hardy only did four hours of mathematics a day. David

did not think it was because Hardy was lazy. It was simply because doing any more mathematics would create stress and fatigue, which stifles creativity, and brings a mathematician closer to the end of his useful life. David did not have such freedom, however. This problem needed to be solved. He would start by writing down the problem once more. It is not that he did not know the problem by now – he knew the statement verbatim. It was that the physical act of writing acted as a trigger, somehow, to the creative processes of his mind. He remembered hearing that the part of the brain that handles mathematics and the part that handles writing were very close together.

It was also true that seeing the parts of the puzzle on the page was very useful. In Algebra, the clean and sterile branch of mathematics, this was especially true. Often, just writing down the problem and the definitions of all concepts was enough. The solution would be right there – you just needed to arrange all the pieces so that, read from top to bottom, it became a proof. However, this particular problem was not an Algebraic one. It was Analysis, which was messy, where there were often too many puzzle pieces to fit on a page (much less in one's mind) and where a crucial, oddly shaped, piece was often missing. This seemed to be the case now. David suspected he was dealing with one of those problems where, if you write down all the steps to get to the answer, each on its own would make sense (at least with a little thought), but upon reaching the end, you would still have no idea how you got there. It was a true mathematician that could understand not only the details of the proof, but the broad outline. It is the latter that is needed to solve a truly knotty problem.

When it turned ten o'clock, it was time to watch an episode of *Caprica*, downloaded via torrent, and then go to bed. No useful mathematics can be done while exhausted. A good eight-hour sleep would bring his mind to that well-rested state needed for clear thinking. It has been shown that sleep is conducive to problem solving – and indeed, many a time a problem that seemed intractable at bedtime was solved by morning. However, David's sleep was never as peaceful as he would like, his mind unable to let the problem be. For this reason, his sleep had to be long. Many students can get by with only five or six hours of sleep. A maths student, at least one that is serious about mathematics, cannot. Now he had more to worry about than just maths, namely the fact that he had not yet found a new place to stay. There was still enough time, however. As he lay his head down he mentally cursed the scoundrels that had tried to swindle him.

Lesson 2

It was yet another rainy morning in Amsterdam. Grey clouds stretched across the sky and rained gloom on all beneath them. David was cycling to class. He knew the route well by now. This allowed his mind to wander and, as always, when a problem remained unsolved, it wandered there. An unsolved problem is a little storm cloud, hovering just above a mathematician's head, always following him around. It was a far greater source of discomfort than

the Dutch weather. David imagined there must have been mathematicians who had seriously attempted to solve a great problem and spent their whole lives living with the storm cloud he is now experiencing. What did the Littlewood experience when his supervisor told him the Riemann Hypothesis seemed like an interesting thesis topic? Probably unknowingly, he attempted to solve one of the most difficult problems in the history of mathematics, still unsolved after hundreds of years. Of course, the storm cloud is not all there is to mathematics. The moment of solving a problem, of seeing and understanding and, especially, of discovering a truly deep and beautiful result brings an ecstasy like that Olympic medallists must experience at the moment of victory or skydivers at the moment their feet tread only air. It is the drug that mathematicians chase. Its effects may last only a moment, because behind each problem, there always lies another, with the same tempting promise.

It was in this manner, only half his mind on physical reality, that David started crossing a street that he had crossed several times before, following the lead of other, presumably native Dutch, cyclists. It was only David's instinct that became aware of the tram coming from the left. And it was not really his conscious mind that managed to step backward on the pedals, braking the bike, but not fast enough. He had to turn as well in order to avoid trundling onto the tram lines and into the path of the oncoming tram. The cyclists ahead of him had purposefully crossed in front of the tram, confident of being able to cross in time, which they narrowly managed to do. Adrenalin having pushed all thoughts of maths out of his head, David cursed the Dutch for their wilful recklessness, trams for being death traps for cyclists, and himself for being so careless. If Brouwer, possibly the only famous Dutch mathematician, had lived in Amsterdam, he would have been run over by a tram, not a car. David thought of mathematicians whose lives had proved to be acutely finite, such as Evariste Galois, who had stupidly managed to die in a duel before the age of twenty, possibly defending the honour of a woman. *The honour of a woman!* His work was so influential as to spawn a new field of mathematics, named after him. His death must have set mathematics back a hundred years, if not more. Of course David knew his own death would probably cost mathematics little. He had not yet even –

“David, why are you standing in the middle of the road?” a voice asked. It was Gerrit, a classmate, on his way to the very same class as David. He spoke with a Dutch accent and was a gangly giant compared to David.

“Just crossing the road,” David answered wryly. “Let's get to class.” He got on his bike, limbs still trembling and cycled beside Gerrit.

“How is that problem set going?” Gerrit asked after a while.

“Still stuck on the last one,” David admitted.

“Yeah, it's tricky.”

It sounded as if Gerrit had found the solution. David was surprised. Gerrit was intelligent, but David was clearly his mathematical superior. The problem peeved him even more now.

“Did you get it then?” David asked.

“Well, no, not really. I found some things online. I can give you the link.”

Ah, so that is it! David thought. “Maybe later,” David answered, “I still want to give it a bash on my own.”

Seeing the solution now would bring no satisfaction. It would mean having failed, that all the hours of thinking had been in vain, and more than that, that his intellect was not incapable of such elementary mathematical thought. No, David would not be Googling the solution until much nearer the deadline.

Lesson 3

As David had expected, being in a tram was only slight more conducive to doing mathematics than nearly being run over by one. However, this was a long tram ride and he was determined not to spend it looking out the window at the rain. It was the rain and the long distance he needed to go that had induced him to take the tram instead of the bike. Large blue letters on a building pronounced “God roept u” in Dutch and next to it, in English, the non-translation “Jesus loves you”. The unmistakeable signage was annoying at first; now it was just part of the landscape, making no more of an impression than filthy open-air urinals in *De Wallen*, the red-light district.

David doubted the signage was particularly effective, not in this enlightened city. It might, however, confuse some tourists interested in Dutch. David, having spent some time in Amsterdam now, had at least learnt some rudimentary Dutch expressions. However, he did not consider learning to speak to Dutch fluently, or even haltingly, to be a productive use of his time. He had no linguistic ambitions and Dutch was neither a very beautiful nor a very useful language. Even inside the Netherlands, most people spoke English and so it was hardly ever needed.

David had gotten over his annoyance at the religious pronouncements and symbols that littered the Dutch landscape. Old churches, some still in use, were still to be found in the *Centrum*. Many considered them beautiful. David considered them to be ostentatious reminders of the excesses and abuses of religion. He was not annoyed because he knew they were merely anachronisms – legacies of the country’s past. The Netherlands was losing religion. Just a few months earlier David had read an article about churches across the country being sold due to a lack of congregants. The congregants that still remained and the small group of Christian students at the university, evidenced only by a few posters advertising Bible studies, were also anachronisms.

David turned his thoughts aside from these anachronisms and focussed on the paper in front of him. It was covered in symbols and scribbles, some horizontal, some vertical. Dots and arrows filled in blank spaces and connected different ideas. Only David could understand what was scrawled on that paper – even other mathematicians would have difficulty. Unconsciously he started chewing the already well-chewed pen he was using. David wondered if Riemann’s notes had looked anything like his. There were no great mathematical revelations in David’s work, but Riemann’s notes may well have contained clues to solving, for instance, the Riemann Hypothesis. We would never know because most of

his notes had been burnt by his housekeeper shortly after Riemann's premature death. She must have looked at those papers and seen nothing but gibberish; she was just tidying up. If any part of Riemann's spirit remained, his soul, one could pray to it, as Jesus did *Riemann, forgive her for she knows not what she is doing*. Of course all that remains of Riemann is his legacy. No soul, no spirit. David desired such a legacy, but even a small footnote in the annals of mathematics would be enough to justify his existence.

"What are you doing?" a voice asked, startling David out of his reverie.

It was the girl next to him. He had hardly noticed her. She was definitely not local. Eastern European, David thought, though he had been wrong about such conclusions several times before.

"Mathematics," David answered, glancing sideways at her, "trying to solve a little problem."

"Can you explain it to me?" she asked.

She seemed genuinely interested, but David could not fathom why. "Do you know any mathematics?" David asked.

"No. Closest I come is reading my credit card bill." She nudged a shopping bag at her feet. "Is it impossible to explain otherwise?"

"Well, let us see. I can give an indication, possibly." David was somewhat flattered by the attention. No non-mathematician ever asked about the *content* of mathematics – they just seemed to think it was too difficult or too boring to bother with. He tried to explain how measures were a way of assigning sizes to things, that he was trying to prove a relationship between two measures using random variables. He did not bother with the technical definition of random variables – to most lay people it was enough to say that it was a variable that took on different values under different circumstances, like the roll of a die. It was unsatisfactory *hand waving* – which means it was not precise and contained no real arguments. Even though the girl nodded, it was clear she did not understand.

"You are very clever," the girl said, "you will find the answer."

David tried not to sigh out loud. This was the default response of non-mathematicians: *you are very clever*. In fact, it was not even true that one needed to be very clever to do mathematics – to do ground breaking mathematics, yes, but not to appreciate the work of others. One just needed to be taught actual mathematics, rather than arithmetic, taught to enjoy the reasoning process rather than fear it.

"Not really," David answered, "it takes practice and patience, like anything else." Lots of patience.

Lesson 4

It was an uncharacteristically sunny day. David was not sure if the happiness he saw on people's faces as he cycled was really due to the sun, or if it was just him projecting his own more cheerful mood (despite not yet having solved the problem he was working on). The sun was a cause for celebration. David

stopped at a red traffic light, allowing other riders not as concerned about traffic rules to overtake him. There was a multitude of stickers on the traffic light pole, many expressing disgust at the consumption of meat. One of the great geniuses, Ramanujan, had been a vegetarian, but probably only because of his Hindu upbringing. Apparently Pythagoras was too, but it seemed unlikely we could know anything about that near-mythical figure with great certainty.

David reached his destination, a little café on a corner. Ilse was already there, looking out over the *gracht*. David parked his bike nearby, making sure to secure both the front wheel and the frame with a chain lock. Ilse smiled as she saw him approach. She had already ordered something, probably a black coffee, as usual. Ilse was a vegan. Despite the accommodating and green reputation of Dutch society, it appeared not yet to have become a place where vegans could live with ease. There were of course a number of good vegan restaurants and shops, but finding anything vegan in a normal supermarket was near impossible it seemed. David had to admire Ilse's determination. She made much of her food from scratch – this was the only effective way to avoid animal products. She used only organic ingredients and went to much trouble to recycle plastic (the local municipalities only recycled glass and paper). All in all, being vegan was very inefficient.

“Where did this fine weather come from?” David said as he approached.

“I had to sell a part of my soul,” Ilse quipped, in a near-perfect British accent. David was surprised to encounter this when he had first met Ilse. She had been one of the guides during his introduction week and he had mistaken her for another international student. Since then, however, David had encountered numerous Dutchies with either American or British English accents.

“You got a bargain,” David countered.

Ilse was a “spiritual” person, not adhering to any religion, but quite willing to accept the idea of a human soul and possibly even a greater power. Unsurprisingly she was studying art history, or something related. Basically, it seemed Ilse's beliefs were a convenient mishmash of ideas, with no thought given to their coherence. David ordered a *koffie verkeerd*, the Dutch version of café latte and Ilse another black coffee. Not for the first time, David was taken by Ilse's beauty. She was uncharacteristically short for a Dutchy, had long straw-coloured hair and deep blue eyes. He was also flattered by the attention she had given him, despite David being reticent and often terse. They had become friends. David was, however, determined not to get romantically engaged, not with any girl. Like the great inventor Nikola Tesla, David considered it to be too much of a distraction.

David's thoughts of Ilse's beauty evaporated, as they always did, when she took out her bag of tobacco and started to roll a cigarette. For a supposedly health-conscious nation, a surprisingly large number of Dutch people (well, Dutch students, at least) smoked, mostly rolling their own cigarettes in order to reduce the cost.

“Can we switch seats?” David asked. Then, as Ilse lifted her brows in an unspoken question, added, “I would like to be upwind when you smoke.”

Ilse shook her head and smiled. “Is there anyone who puts up with you?”

she asked and got up so they could switch seats.

“My mother, sometimes,” David replied.

The conversation turned to Mathematics, as it often did. “I understand its useful,” Ilse said, “Society needs mathematics.”

“Except it isn’t useful,” David replied. “Some small amount of mathematics has turned out to be useful, but the majority of modern mathematics seems to have hardly any use at all. The most interesting mathematics is often the most useless. Sometimes, surprisingly, our results do turn out to be useful, but that is not why we do maths.”

“Why do it then?”

“Because it is beautiful and because it reveals deep truths. Nothing can captivate my mind quite like mathematics. It is not unlike creating or admiring a work of art.”

“So you create mathematics?”

“Well, perhaps. Either we create it like a painting, or we discover it, like physicists discover physical laws. Perhaps it is both. The distinction may be meaningless.”

“How is maths beautiful?”

David thought for a second, realised it was hard to explain. “Mathematics can connect many deep ideas, reveal mysteries. Proofs can be surprisingly short and elegant. Sometimes results can have far-reaching consequences, which could never be foreseen. Maths organises human thinking, which can be fuzzy and messy, and makes it precise and clean, so its full potential can be developed.”

Ilse’s face betrayed that she still did not grasp it. Of course, she’s only ever really done arithmetic, so how could she?

“Let me give an example,” David said and rummaged in his bag for a scrap of paper.

“Do you always carry pen and paper around?” Ilse asked.

“Of course,” David answered simply, without any further explanation. On the paper he wrote an equation:

$$e^{i\pi} + 1 = 0.$$

He turned the page so Ilse could see it clearly. “This is the Euler identity,” David explained, “and it is considered to be one of the most beautiful identities in mathematics. It contains the very special numbers e , π and zero, 1, and i , which is the square root of minus 1. It contains the operations addition, multiplication and exponentiation. So many links in one line.”

Ilse smiled, clearly amused. She started to speak, but then David turned the piece of paper around and started writing something else. “You have something else to show me?” she asked.

Not looking up, David answered, “No, sorry, I just thought of an idea.” Ideas were sometimes fleeting and had to be written down before they were lost. It could be a breakthrough. After a few minutes he looked up. “So, do you understand?” he asked.

Ilse shook her head, “I don’t think I’ll ever understand.”

Lesson 5

“Are you the one putting up the posters now?” David asked.

The girl turned. “Just today,” she said, her smile a little less broad, her eyes a little narrower, “doing a favour.”

“Does anyone ever respond?” David asked.

“I did,” she said and moved past him, revealing the poster advertising a Christian student group. “You are, as always, welcome to join us.” The smile was broad once more. “Also, there’s an organ concert at De Oude Kerk on Saturday. I might go. You like organ music, right?”

“I do, but I’d rather not go. But you knew that.”

“You could always change your mind.”

She was referring to David’s refusal to enter a church, any church. They had been in a group visiting various monuments in the *Centrum*, including De Oude Kerk, and David had refused to enter any of the churches, not even the ones which were now merely museums. David could not help feeling like Hardy, who had also refused to enter places of worship. It felt dignified, defiant. Perhaps it was vain, but he would not change his mind now, not for a homely and irrational English girl.

“Lunch?” he asked.

“Certainly,” she answered, “Let me just hang up this last poster.”

Michelle was one of only a handful of girls and the only Christian in David’s programme. She was annoyingly enthusiastic, always asking questions. She caught up with David in the cafeteria and piled mostly salad onto her tray. David wondered whether she was on a diet or merely a tight budget.

“Have you managed to do that last problem on the problem sheet for next week?” she asked as they walked to a table. “I’m rather stuck.”

“Almost there,” David replies. The idea he got while talking to Ilse had turned out to be useful. “You should talk to Gerrit. He found something on the net.”

“Ah.”

“Or you could pray...”

“I do pray,” she said, still smiling. “You should try it.”

“I would rather solve it on my own.”

It was, of course, not true that mathematicians, good mathematicians, that is, cannot be religious. Many famous mathematicians in history have been religious. Cauchy was a devout Catholic, annoyingly so, as his views often alienated him from his colleagues. But ever since Bertrand Russell, with the development of modern, rigorous mathematics, the idea of God had seemed to become an absurdity. David found it strange that a mathematician could accept the rigour of modern mathematics, the strict requirements for truth, and still choose to believe in a mythical being. At least Michelle was not a particularly harmful Christian – she did not seem to be unquestioning and dogmatic. But still, she bought into the ridiculous notion that being homosexual was somehow sinful. How could such blindness accompany the clarity of mathematical thought?

“You should read something by Dawkins or *Why I am not Christian* by Bertrand Russell.”

“I should,” she agreed. “But I admit, I am a little scared to. Men with great minds can argue falsely quite convincingly.”

“If they argue with logic, surely you have nothing to fear but the truth.”

“Only in mathematics is the truth clearly the truth. In everything else...” She was not smiling now. For a moment her thoughts turned inward, her eyes ceased to look at anything. Then the smile returned, her eyes looked straight at him once more; the usual glimmer was there again.

“You always make me think,” she said.

Lesson 6

David looked at the aisles of food in the small supermarket and wondered what he would do with a larger selection. He had already been here for half an hour and had spent most of that time deliberating on what to buy. His basket contained only a few items. Outside it was overcast. At the entrance stood a man selling *Z* magazines, talking to a woman, probably a housewife. In reality, perhaps, this latter service, being someone to talk to for housewives, was the man’s real source of profit.

David turned a corner. He knew he should just pick something. But it might be the wrong thing, like when Grothendieck was asked to pick a prime number and picked 57 (which is patently not prime), now known as *Grothendieck’s prime*. Grothendieck, being brilliant, became the more famous for his gaffe (if indeed, it was not done intentionally). There was no fame to be gained here. David picked an item and started for the tills. But something was wrong. He heard shouting. At the tills stood a man in a ski mask, shouting and waving. He was shouting loudly, but somehow David could not hear him. It was like he had gone deaf. The man had something in his hand. A gun? No, a knife. David was still out of sight and stepped back into the aisle. He pressed his back against the shelving and felt his heart pound. Blood rushed to his head.

Somewhere beneath the rushing blood, David thoughts turned to mathematicians involved in life or death situations. Littlewood, no, he only did projectile calculations during WWI. Volterra fought at the front lines though. André Weil, arrested on suspicion of being a spy during the Winter War. Jean Cavallès, died as a member of the French resistance during WWII. Galois, a pistol pointed at him, willing to die, for nothing. David felt that pistol point at himself. Galois was holding the pistol, right in his face. He was saying something. David could not hear him.

Slowly sound returned. There was a man in front of him. “Sir, are you ok? The man is gone. The police are coming.”

David nodded, slowly.

“Wait here,” the man said.

Amsterdam was supposed to be a safe country. The place where David could live his out his dreams. Ramanujan was not meant to die of some unknown

disease in England, just a few years after his arrival, just when he had become famous and it was clear a life of mathematical pioneering lay before him. Your dreams should not cost you your life. At least Ramanujan tasted his dreams – he left an indelible mark on mathematical history, even with only a handful of publications. David could not say that. He could not die, not yet. He walked through the aisles, past the tills. He still had his basket with his groceries, but no one stopped him. The tills were all open, all the cash gone.

“*Kaaskop!*” a cyclist yelled when David crossed the cycle lane in front of him.

The man narrowly avoided hitting David. David reached his building, not knowing how he had got there. He vaguely remembered having left with his bike, so it must still be at the shop. Death had thought about death often. Now it seemed to invade his mind. For most people death is true nothingness. A few people remember your life, then they die, then the records of your life are buried, forgotten or destroyed. And nothing remains. A mathematician can, in a sense, avoid death, through his work. Every contribution, even if only modest, is taken up, added to, used, reinvented. After thousands of years, do we not still admire Euclid and Archimedes, though their methods were ancient, their understanding imperfect?

But still, death could not be overcome. There was no afterlife, no heaven, no realm of the gods. Of all the things David had heard about WWII, few things really made an impression, except one, the death of the mathematician Hausdorff. He and his wife, after being sent to a concentration camp, together committed suicide. Many people commit suicide, of course, and many did so during WWII. For a mathematician to do so, under normal circumstances, would be an atrocity on its own. It betrays a selfishness, an utter disregard for mathematics, to willingly annihilate all one’s possible future contributions. But to place a mathematician in a situation so vile, so unbearable, to force him to do so.... Somehow that one death, not even directly by Nazi hands, seemed more horrible, more despicable, than any other act the Nazis committed.

Its not WWII. No Nazis. Just a robbery. No deaths. I am alive, David reminded himself. He put his free hand in his pocket, looking for his keys. Instead he pulled out his phone. *I should call someone,* he thought, *but who?*